

Linear Modulation of the Resonant Frequency of a Double-bubble Model Below an Air–Water Interface

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Abstract—The study of sound wave propagation across the air–water interface is important for several potential applications, e.g., acoustic communication between underwater environments and air. However, only 0.1% of the power of the incident wave perpendicular to the interface can cross the interface owing to the large acoustic-impedance mismatch between air and water. This renders the propagation of acoustic energy between the media inefficient, leading to the failure of effective acoustic communication between these media. This situation could be solved using designed bubble systems because bubbles are good acoustic impedance matchers. The existing research is mainly limited periodic single-bubble systems, which are not suitable for acoustic propagation in the broadband frequency range. This is because the large sound transmission coefficient between the air–water interface occurs only near a sharp resonant frequency and decreases rapidly away from the resonance. In this work, we propose a periodic double-bubble model below the air–water interface for efficient propagation over a wide frequency range. A numerical simulation model based on COMSOL was developed and validated through comparison with a theoretically analyzed periodic single-bubble system. Then, a periodic double-bubble model was built to study the modulation effect of the resonance frequency versus the vertical interval between the two bubbles. Numerical simulations show that the resonant frequency of the periodic double-bubble system varies linearly with the vertical interval. Therefore this study offers a potential method for designing broadband acoustic communication techniques across the sea–air interface.

Keywords—double-bubble model, air–water interface, acoustic impedance matcher, sound transmission coefficient, marine engineering, COMSOL

I. INTRODUCTION

It has been attracting increasing attention to obtain timely underwater information from the air using trans-medium communication technology [1, 2]. This technology can be employed to explore marine organisms and the underwater environment efficiently [3, 4], locate underwater equipment precisely, and detect breakdowns in maritime facilities conveniently [5]. There are several ways to realize

information interaction between the sea and air. A recent method is to combine the transmission of underwater sound and airborne electromagnetic waves to achieve communication across mediums. The process is divided into three steps. First, underwater sound waves excite vibration on the water surface. Then, the airborne radar probes the displacement of water surface vibrations and decodes it. Finally, information transmitted by underwater sound waves can be obtained, and information interaction between the sea and air can be achieved [6]. However, this approach has some limitations. For example, it is inapplicable under rough sea conditions because radar sensors cannot measure the correct displacement on a randomly curved surface. Additionally, the transmitting acoustic transducer and receiving radar sensor must work vertically, which is not always the case in engineering.

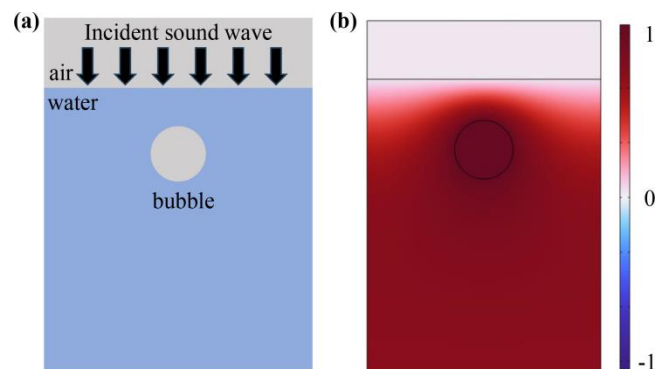


Fig. 1. The propagation of sound waves across the air–water interface with a bubble below. (a) A unit of the periodic single-bubble system. (b) The normalized pressure of sound propagation from air to water at the resonance frequency.

One possible solution to achieve the information interaction across the sea–air interface is to use bubbles as an acoustic impedance coupler of air–water media for sound propagation, which enables efficient transmission of sound energy across media [7]. The coupler model of the bubble resonance is shown in Fig. 1(a). Fig. 1(b) shows the normalized pressure of sound propagation from air to water at the resonance frequency of the single-bubble system. As

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shown, an underwater bubble is a relatively good coupler (also called acoustic impedance matcher). However, high transmission efficiency occurs only near the narrow resonance frequency band, which is not good for broadband communication. To overcome this shortcoming, this paper proposes a periodic double-bubble model that precisely modulates the resonance frequency and extends the frequency range for potential applications in acoustic communication.

II. PERIODIC SINGLE-BUBBLE SYSTEM

As we know, there is a large difference in the acoustic impedance between air and water. Specifically, the acoustic impedance of water is 3,600 times that of air, resulting in a transmission coefficient of sound intensity between air and water <1% (as shown in Fig. 2(c)) [8, 9]. Thus, effective sound propagation between air and water is very difficult, and most sound waves are reflected back at the air–water interface. It is not possible to employ sound wave carriers to efficiently interact with information in water and air environments. To solve this issue, recent studies provide one possible way that uses the periodic single-bubble system below the air–water interface [see a unit shown in Fig. 1(a)]. The bubbles can be taken as the acoustic impedance matchers of air and water media, which significantly increase the transmission coefficient of sound waves [7]. Hence, bubble can achieve broadband communication over the sea and air.

A. Radiation Model for Periodic Single-bubble System

Next, we describe the theoretical analysis of the periodic single-bubble system, which is treated as a two-dimensional lossless case. Fig. 2(a) shows the sound radiation model of a single bubble. It is assumed that monopole sound sources excite a bubble to vibrate asymmetrically to radiate energy into air and water with the to-air radiation coefficient γ_a and to-water radiation coefficient γ_w [10]. The bubble is excited to resonate with monopole sound sources. To obtain the to-air and to-water radiation coefficients and the resonance angle frequency ω_0 of the bubble, numerical simulations of the acoustic radiation under a sweeping frequency were conducted using the commercial software COMSOL6.1. Through the simulation calculation, the damping components of c_a and c_w in the asymmetrical vibration of the bubble and the effective mass m_e of the bubble can be obtained. Finally, the radiation coefficients and angle frequency can be obtained through an acoustic radiation model, as described in [10], where $\gamma_a = c_a/m_e$, $\gamma_w = c_w/m_e$, and $\omega_0 = \sqrt{k/m_e}$. k is the stiffness of the bubble, and m_e is the effective mass of the bubble caused by the inertia effect of the fluid around the bubble. In this way, the parameters of the acoustic propagation model can be obtained.

B. Acoustic Propagation Model for Periodic Single-bubble System

After obtaining the parameters of the sound radiation model, we performed a theoretical analysis of the sound propagation model of the periodic single-bubble system. The size of the periodic single-bubble system was set as the water depth of the bubble $t = 7$ mm, the width in one unit

$D = 40$ mm, and the radius of the bubble $R = 5$ mm, which are the critical coupling conditions described in [10]. The sound (power) transmission coefficient is the ratio of the transmitted power to the incident power given by (1) [10]. Because the asymmetrical vibration of bubbles with the same external condition and size excited by incident sound waves and monopole sound sources with the same frequency are identical, the radiation coefficients γ_a and γ_w of the bubbles under two different excitations are equal. We can bring the calculated radiation coefficients γ_a and γ_w , and the resonance angle frequency ω_0 of the bubble in the sound radiation model into (1) of the sound propagation model with the same size. Hence, the corresponding transmission coefficient of the sound propagation model can be computed on the basis of (1), which shows that when the sound wave of one frequency can excite a bubble to resonate, its transmission coefficient will be the largest.

$$T(\omega) = \gamma_w \gamma_a \left| \frac{\omega}{(\gamma_a + \gamma_w)\omega i - \omega^2 + \omega_0^2} \right|^2 |T_0^2| \quad (1)$$

where T is the transmission coefficient of the sound intensity, ω is the frequency of incident sound waves, and $T_0 = 2Z_w/(Z_a + Z_w) \approx 2$ is the transmission coefficient from air to water without the bubble, in which Z_a and Z_w are the acoustic impedances of air and water, respectively.

According to (1), we obtain the transmission coefficient curve of incident sound waves with different frequencies corresponding to the periodic single-bubble system, as shown in the analytical curve in Fig. 2(d). From the curve, it can be verified that the bubble underwater is an excellent acoustic impedance matcher with excellent performance.

C. Numerical Simulation for Periodic Single-bubble System

We developed a numerical simulation model of the same periodic single-bubble system as that used in the theoretical analysis in COMSOL6.1, as shown in Fig. 2(b). Through simulation calculations, the transmission coefficient curves of incident sound waves with different frequencies under the periodic single-bubble system can be obtained, indicated by the numerical curve shown in Fig. 2(d).

The concrete modeling process for the numerical simulation model is divided into two parts. The first part is the boundary condition of the model. As shown in Fig. 2(b), the model contains five regions. The left and right sides of the air and water regions were set as periodic boundary conditions that consider an array of bubbles with a period D [10]. The bubble region contains air without boundary conditions. The two remaining regions lying at the top and bottom of the model were set as the perfect matching layers (PMLs) with the 30th width of the incident sound wavelength. Another is the mesh generation. The mesh sizes of the water, air, and bubble regions is calibrated to be more detailed under the meaning of fluid dynamics, and the mesh type is the free triangle mesh. Meanwhile, the mesh size of PML with the free quadrilateral mesh was set to 0.5 mm, and it was divided into mapping units. Overall, the sum of the two types of mesh was 6,058 units.

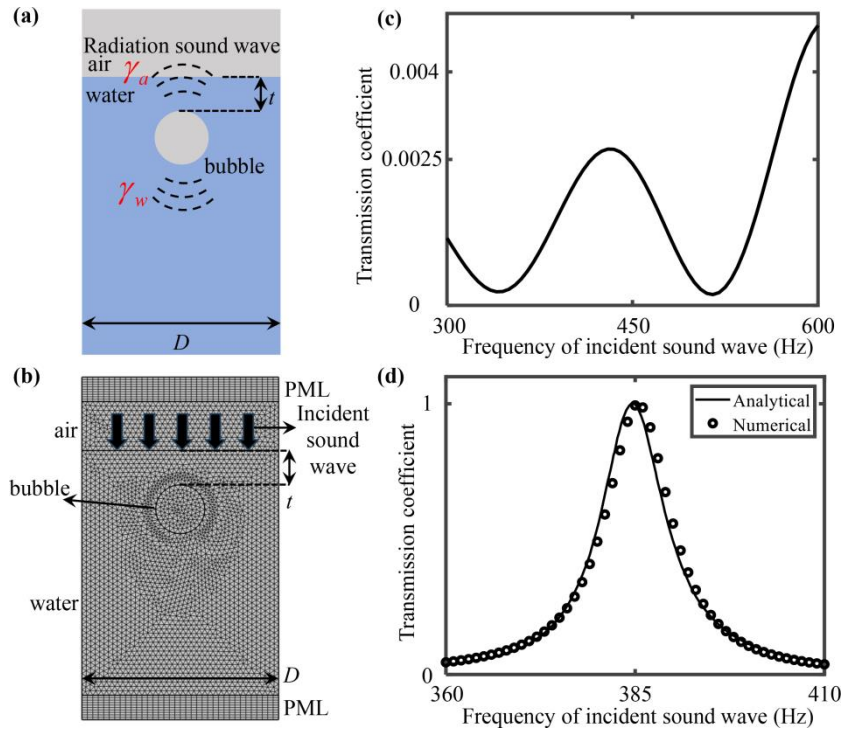


Fig. 2. (a) The sound radiation model of a bubble below the air–water interface in a unit. D stands for the width of the water field in a unit, which is 40 mm. $t = 7$ mm is the depth of the bubble below the interface. The bubble in this model contains monopole sound sources that excite the bubble to vibrate asymmetrically, and the radius of the bubble is set as 5 mm. (b) The mesh model of a unit of the periodic single-bubble system in COMSOL6.1. (c) The transmission coefficient of the sound intensity from the air to the water versus the incident frequency without a bubble. (d) Transmission coefficient curves of incident sound waves with different frequencies under the same periodic single-bubble system. The analytical curve is obtained by the theoretical analysis extracted from [10] and the numerical curve indicated by circles is the numerical results from COMSOL 6.1.

The transmission coefficient curves of incident sound waves with different frequencies under the same periodic single-bubble system were studied, as shown in Fig. 2(d). The analytical curve was obtained using the theoretical analysis extracted from [10], and the numerical results indicated by circles were computed using COMSOL 6.1. Our simulation results agreed well with the theoretical results, demonstrating the effectiveness of the numerical simulation model.

However, Fig. 2(d) shows that the transmission coefficient of incident sound waves is relatively high only when their frequencies are near the sharp resonant frequency, and when they are far from the resonant frequency, their transmission coefficients decrease rapidly. Thus, in a periodic single-bubble system, when a band signal propagates into the water, the sound signal generates selective debilitation, which is not conducive to the propagation of broadband sound energy across water and air media.

III. RESEARCH FOR MODULATING THE RESONANCE FREQUENCY PRECISELY

A single bubble as the acoustic impedance matcher is suitable only for efficient sound propagation in sharp bands. Broadband communication across media forms the foundation of information interaction across the sea and air. To achieve this, we developed a periodic double-bubble model underwater near the interface (Fig. 3(a)). Then, on the basis of this model, we studied the influence of the double-bubble combination underwater as an acoustic impedance matcher for the transmission coefficient of incident sound waves with different frequencies, new changes, and the laws of its influence compared with that of a single bubble.

Meanwhile, we also conducted an intensive research on the modulation law with the vertical interval between two bubbles of the resonance frequency of the double-bubble model underwater.

The numerical simulation model of the proposed periodic single-bubble system was confirmed to be correct. Therefore, we applied the modeling method of this simulation model to build a similar simulation model for the periodic double-bubble model underwater near the proposed interface, as shown in Fig. 3(b). By performing simulation calculations for the mesh model of periodic double-bubble systems with different vertical intervals $h1$ in COMSOL6.1, the corresponding transmission coefficient curves of incident sound waves with different frequencies can be obtained, as shown in Fig. 3(c). Fig. 3(c) shows that the influence of the double-bubble combination underwater on the transmission coefficient of incident sound waves with different frequencies was similar to that of a single bubble. The corresponding transmission coefficient curves only generate skewing, and the peak values of the curves change only slightly, which still corresponds to the resonance frequency of double-bubble combinations in periodic double-bubble systems.

Then, we further analyzed the relation between the resonance frequencies corresponding to the peak values of the transmission coefficient curves and vertical intervals of the double bubbles in periodic double-bubble systems. It can be seen that the resonance frequency of the periodic double-bubble system exhibits a linear decreasing change with the vertical interval of the corresponding double bubbles, as shown in Fig. 3(d). Thus, we can precisely modulate the inherent frequency of a periodic double-bubble system according to this law, providing a potential method for

designing the broadband acoustic communication techniques

across the sea–air interface.

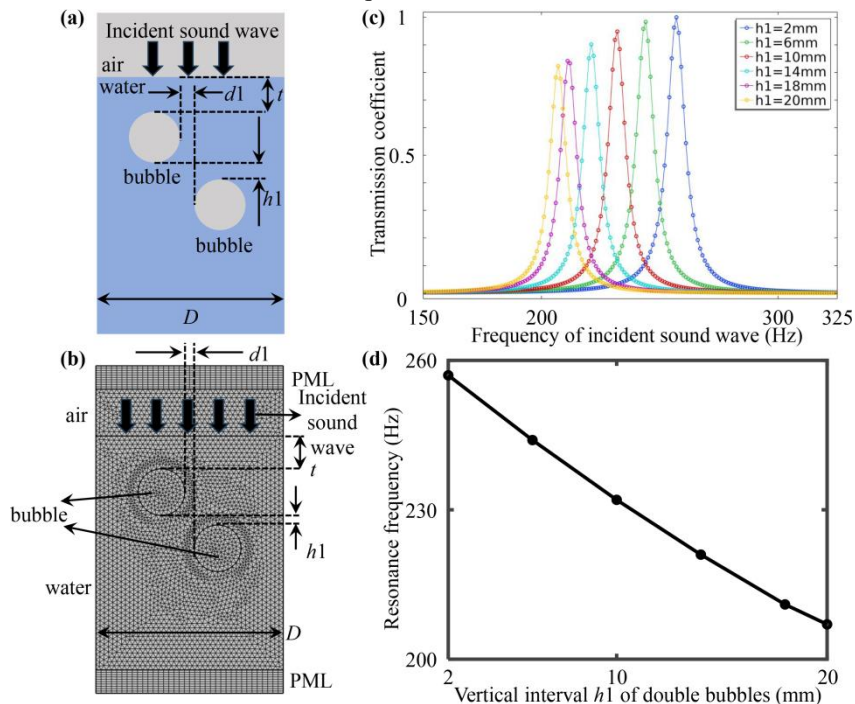


Fig. 3. (a) A unit of the periodic double-bubble model underwater near the air–water interface. D stands for the width of the water field in a unit, which is 40 mm. $t = 7$ mm is the depth of double bubbles. The horizontal interval $d1$ of two bubbles in this model is 2 mm, and the vertical interval $h1$ of two bubbles in this model is changeable. (b) The mesh model of a unit of the periodic double-bubble system with the vertical interval $h1$ changeable in COMSOL6.1. (c) The transmission coefficient versus the frequency of incident sound waves for periodic double-bubble systems with different vertical intervals $h1$. (d) The linear relationship of the resonance frequency versus the vertical interval between two bubbles.

IV. CONCLUSIONS AND OUTLOOK

This study proposes a double-bubble model below the air–water interface to precisely tune resonance frequencies. Numerical simulations demonstrate that the resonant frequency varies with the vertical interval between the two bubbles. The double-bubble model overcomes the disadvantage of the single-bubble model, which only has a large transmission coefficient at or near the sharp resonance frequency. By combining the double-bubble models with different vertical intervals for different units, the designed system can enhance the acoustic transmission between air and water for a broad range of frequencies. Although this model can provide more degrees of freedom and a better ability to tune the resonance frequencies for broadband acoustic propagation across the interface, the engineering implementation to produce and confine the bubbles in a good manner is challenging. In the future, we will focus on general models of equivalent bubble systems that can be easily fabricated and modulated for a broad range of resonance frequencies.

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